Autovalores y autovectores.

Pag 3.-

$$(x\ \ y)\begin{pmatrix}a & h\\ h & b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = (x\ \ y)\begin{pmatrix}ax+hy\\ hx+by\end{pmatrix} = ax^2+hxy+hxy+by^2$$

$$= ax^2+2hxy+by^2$$

Pag 22.- Ruffini

1	1	-21	-45
5	5	30	45
1	6	9	0

$$\lambda^3+\lambda^2-21\lambda-45 = (\lambda-5)(\lambda^2+6\lambda+9)$$
$$= (\lambda-5)(\lambda+3)^2$$

Continuando Pag 22

$$\begin{pmatrix}-7 & 2 & -3\\ 2 & -4 & -6\\ -1 & -2 & -5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \longrightarrow \begin{array}{l} -7x_1+2x_2-3x_3=0\\ 2x_1-4x_2-6x_3=0\\ -x_1-2x_2-5x_3=0\end{array} \longrightarrow x_1=-2x_2-5x_3$$

$x_2=-2x_3$

$-x_2-2x_3=0$

$-8x_2-16x_3=0$

$-4x_2-10x_3-4x_2-6x_3=0$

$x_1=-2x_2-5x_3$

$x_1=4x_3-5x_3$

$x_1=-x_3$

si $x_3=\alpha \Rightarrow x_1=\alpha$ y $x_2=2\alpha$

$$\begin{pmatrix}\alpha\\ 2\alpha\\ -\alpha\end{pmatrix} \Rightarrow \alpha\begin{pmatrix}1\\ 2\\ -1\end{pmatrix}$$

$$\begin{pmatrix}1 & 2 & -3\\ 2 & 4 & -6\\ -1 & -2 & 3\end{pmatrix} \xrightarrow[f_3+f_1]{f_2-f_1(2)} \begin{pmatrix}1 & 2 & -3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

Pag 16 $y'_1 = (a+b)\,y_1$ $\boxed{\text{Ec. dif}}$

$$y'_1 = (a+b)\,\frac{(x_1+x_2)}{2} = \frac{ax_1+ax_2+bx_1+bx_2}{2} = \frac{x_1(a+b)+x_2(a+b)}{2}$$

$\boxed{\text{Ec dif}}$

$$M(t) = \begin{vmatrix} e^{(a+b)t} & e^{(a-b)t}\\ e^{(a+b)t} & -e^{(a-b)t}\end{vmatrix} = -e^{at+\cancel{bt}+at-\cancel{bt}} - e^{at-\cancel{bt}+at+\cancel{bt}}$$

$$-e^{2at}-e^{2at} = -2e^{2at}$$

$$\left(e^{\alpha t}\cos(\beta t)+e^{\alpha t}i\,\text{sen}(\beta t)\right)(\mu+i\omega)$$

$$e^{\alpha t}\cos(\beta t)\,\mu + e^{\alpha t}i\,\text{sen}(\beta t)\,\mu + e^{\alpha t}\cos(\beta t)\,i\omega + e^{\alpha t}i\,\text{sen}(\beta t)\,i\omega$$

$$e^{\alpha t}\mu\cos(\beta t) + i e^{\alpha t}\left[\omega\cos(\beta t)+\mu\,\text{sen}(\beta t)\right] + e^{\alpha t}\underbrace{i^2}_{-1}\text{sen}(\beta t)\,\omega$$

$$e^{\alpha t}\left[\mu\cos(\beta t)-\omega\,\text{sen}(\beta t)\right] + i e^{\alpha t}\left[\omega\cos(\beta t)+\mu\,\text{sen}(\beta t)\right]$$

$2a + 4at + 2b = t$

$4at + (2a + 2b) = 1t + 0$

$4a = 1$ $\qquad$ $2a + 2b = 0$

$a = 1/4$ $\qquad$ $2b = -1/2$

$b = -1/4$

$\frac{1}{4} t^2 e^t + \left(-\frac{1}{4}\right) t e^t$ $\qquad$ $\frac{1}{4} t^2 e^t - \frac{1}{4} t e^t = y_p$

$2 \cdot \frac{1}{4} + \cancel{4} \frac{1}{\cancel{4}} t + 0 = t$ $\qquad$ $y_{HH} + y_p = y_{nH}$

$\frac{1}{2} + t = t$

$e^{\lambda t}$

$$\boxed{y_{nH} = e^{t} - e^{-t} + \frac{1}{4} t^2 e^t - \frac{1}{4} t e^t}$$

λe^t

c_1

$\lambda_1 \mu_1 + \lambda_2 \mu_2$

$e^{\lambda t}$

Consulta Previa al Parcial.

$X'(t) = A\,X(t)$, siendo $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$ y $A = \begin{pmatrix} -2 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}$

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{pmatrix} = X'(t) = \begin{pmatrix} -2 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{pmatrix} -2x_1(t) \\ -x_1(t) \\ x_1(t) + x_2(t) - x_3(t) \end{pmatrix}$$

$$\exists\ X(t) = c_1\, e^{\lambda_1 t} \begin{pmatrix} \\ \\ \end{pmatrix} + c_2\, e^{\lambda_2 t} \begin{pmatrix} \\ \\ \end{pmatrix} + c_3\, e^{\lambda_3 t} \begin{pmatrix} \\ \\ \end{pmatrix}$$

cholesky -

A

n x n

simétrica

def. positiva → λ todos positivos

$$= e^{\alpha t}\ e^{i\beta t}$$
$$= e^{\alpha t}[\cos(\beta t) + i\,\mathrm{sen}(\beta t)]$$

Ec. dif.

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 e^{(a+ib)t}\begin{pmatrix} 1 \\ i \end{pmatrix} + c_2 e^{(a-ib)t}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\text{si}\begin{cases} e^{at}e^{ibt} = e^{at}[\cos(bt) + i\,\mathrm{sen}(bt)] \\ e^{at}e^{-ibt} = e^{at}[\cos(bt) - i\,\mathrm{sen}(bt)] \end{cases}$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 e^{at}[\cos(bt) + i\,\mathrm{sen}(bt)]\begin{pmatrix} 1 \\ i \end{pmatrix} + c_2 e^{at}[\cos(bt) - i\,\mathrm{sen}(bt)]\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$= \begin{pmatrix} c_1 e^{at}[\cos(bt) + i\,\mathrm{sen}(bt)] + c_2 e^{at}[\cos(bt) - i\,\mathrm{sen}(bt)] \\ i c_1 e^{at}[\cos(bt) + i\,\mathrm{sen}(bt)] - i c_2 e^{at}[\cos(bt) - i\,\mathrm{sen}(bt)] \end{pmatrix}$$

$$= \begin{pmatrix} \underline{c_1 e^{at}\cos(bt)} + i c_1 \mathrm{sen}(bt) e^{at} + \underline{c_2 e^{at}\cos(bt)} - c_2 e^{at} i\,\mathrm{sen}(bt) \\ i c_1 e^{at}\cos(bt) + i^2 c_1 e^{at}\mathrm{sen}(bt) - i c_2 e^{at}\cos(bt) + i^2 c_2 e^{at}\mathrm{sen}(bt) \end{pmatrix}$$

$$= \begin{pmatrix} e^{at}\cos(bt)(c_1 + c_2) + i\,\mathrm{sen}(bt) e^{at}(c_1 - c_2) \\ e^{at}\cos(bt) i (c_1 - c_2) + i^2 e^{at}\mathrm{sen}(bt)(c_1 + c_2) \end{pmatrix}$$

$$= \begin{pmatrix} e^{at}\cos(bt)(c_1 + c_2) + i\,\mathrm{sen}(bt) e^{at}(c_1 - c_2) \\ -e^{at}\mathrm{sen}(bt)(c_1 + c_2) + e^{at}\cos(bt) i (c_1 - c_2) \end{pmatrix}$$

$$= \left(\underbrace{e^{at}(c_1 + c_2)}_{c_1'} \begin{pmatrix} \cos(bt) \\ -\mathrm{sen}(bt) \end{pmatrix} + \underbrace{i e^{at}(c_1 - c_2)}_{c_2'} \begin{pmatrix} \mathrm{sen}(bt) \\ \cos(bt) \end{pmatrix} \right)$$

$$x' = A(t)x + B(t)$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} \left[\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} t + \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \right] + \begin{pmatrix} 6 \\ -10 \end{pmatrix} t + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} a_2 t + a_1 \\ b_2 t + b_1 \end{pmatrix} + \begin{pmatrix} 6t \\ -10t + 4 \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} 6a_2 t + 6a_1 + b_2 t + b_1 + 6t \\ 4a_2 t + 4a_1 + 3b_2 t + 3b_1 - 10t + 4 \end{pmatrix}$$

$$a_2 = t(6a_2 + b_2 + 6) + (6a_1 + b_1)$$
$$b_2 = t(4a_2 + 3b_2 - 10) + (4a_1 + 3b_1 + 4)$$

$$6a_2 + b_2 + 6 = 0 \rightarrow a_2 = -b_2 - 6/6 = -\frac{b_2}{6} - 1$$

$$4a_2 + 3b_2 - 10 = 0 \rightarrow 4\left(-\frac{b_2}{6} - 1\right) + 3b_2 - 10 = 0$$

$$a_2 = -2$$

$$-\frac{2b_2}{3} - 4 + 3b_2 - 10 = 0$$

$$\frac{7}{3} b_2 = 14 \longrightarrow b_2 = 6$$

$$\begin{pmatrix} \cancel{a}-\cancel{a}+ib & b \\ -b & \cancel{a}-\cancel{a}+ib \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} ib x_1 + b x_2 = 0 \rightarrow i x_1 + x_2 = 0 \\ -b x_1 + ib x_2 = 0 \rightarrow -x_1 + i x_2 = 0 \end{matrix} \rightarrow \begin{matrix} x_2 = -i x_1 \\ + x_1 = i x_2 \end{matrix}$$

λ_2

si $x_1 = 1 \rightarrow x_2 = -i$

$$\begin{pmatrix} \cancel{a}-\cancel{a}-ib & b \\ -b & \cancel{a}-\cancel{a}-ib \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} -i\cancel{b}x_1 + \cancel{b}x_2 = 0 \\ -\cancel{b}x_1 - i\cancel{b}x_2 = 0 \end{matrix} \rightarrow \begin{matrix} -i x_1 + x_2 = 0 \\ -x_1 - i x_2 = 0 \end{matrix} \rightarrow \begin{matrix} x_2 = i x_1 \\ -x_1 = i x_2 \end{matrix}$$

$-a_1 + 2b_1 - 8 = 0 \rightarrow -b_1 - 3 + 2b_1 - 8 = 0 \rightarrow b_1 = 11$

$-a_1 + b_1 + 3 = 0 \rightarrow a_1 = b_1 + 3 \rightarrow a_1 = 14$

$\mathbb{R} + \mathbb{I}$

$\frac{1}{x} = \frac{1}{a+bi}$ en teoría sería esto pero tiene que estar

$$x^{-1} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a-bi}{a^2 \cancel{+abi} \cancel{-abi} - b\underbrace{i^2}_{-1}} = \frac{a-bi}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{bi}{a^2+b^2}$$

$$\begin{pmatrix} -1-i & 2 \\ -1 & 1-i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$(-1-i)x_1 + 2x_2 = 0 \rightarrow (1-i)(-1-i) + 2x_2 = 0 \rightarrow 2x_2 = -(1-i)(-1-i)$

$-x_1 + (1-i)x_2 = 0 \rightarrow (1-i)x_2 = x_1$

$2x_2 = -(-1 \cancel{-i} \cancel{+i} + i^2)$

$2x_2 = 1 - i^2$

$\cancel{2}x_2 = \cancel{2} \rightarrow x_2 = 1$

$$\begin{pmatrix} 1-i \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1+i & 2 \\ -1 & 1+i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$(-1+i)x_1 + 2x_2 = 0 \rightarrow (i-1)x_1 + 2x_2 = 0$

$-x_1 + ((1+i)x_2) = 0 \rightarrow (1+i)x_2 = x_1$

$$\begin{pmatrix} 1+i \\ 1 \end{pmatrix}$$

$$(\cos(t) + i\,\mathrm{sen}(t))\begin{pmatrix} 1-i \\ 1 \end{pmatrix} + (\cos(t) - i\,\mathrm{sen}(t))\begin{pmatrix} 1+i \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos t + i\,\mathrm{sen}\,t - i\cos t - i^2\mathrm{sen}\,t \\ \cos t + i\,\mathrm{sen}\,t \end{pmatrix} + \begin{pmatrix} \cos t - i\,\mathrm{sen}\,t + i\cos t - i^2\mathrm{sen}\,t \\ \cos t - i\,\mathrm{sen}\,t \end{pmatrix}$$

$$\begin{pmatrix} \cos t + \mathrm{sen}\,t \cancel{+ i\,\mathrm{sen}\,t} \cancel{- i\cos t} + \cos t \cancel{- i\,\mathrm{sen}\,t} \cancel{+ i\cos t} + \mathrm{sen}\,t \\ \cos t \cancel{+ i\,\mathrm{sen}\,t} + \cos t \cancel{- i\,\mathrm{sen}\,t} \end{pmatrix}$$

$$\begin{pmatrix} 2\cos t + 2\,\mathrm{sen}\,t \\ 2\cos t \end{pmatrix}$$

$AXt + Bt$ $\quad +2 \quad \underline{x_p = at+b}$

$\underline{ate^t + be^t}$

$$2ae^t + (at+b)e^t - 2\left(ae^t + (at+b)e^t\right) + (at+b)e^t = te^t$$

$$2ae^t + ate^t + be^t - 2ae^t - 2ate^t - 2be^t + ate^t + be^t = te^t$$

$$2a + at + b - 2a - 2at - 2b + at + b = t$$

$\cancel{2a = t}$ $\quad 0t + 0 \quad 0 = t$

$2a = 0t + 0$

$\cancel{2a = 0}$ Absurdo

$$y_p = at^2 + bt + c$$

$$y_p' = 2at + b$$

$$y_p'' = 2a$$

$y'' - 4y' = t^2$

$y'' - 4y' = 0.$

$$2a - 4(2at + b) = t^2$$

$$2a - 8at - 4b = t^2$$

$$y'' - 4y' = 0$$

$$2a - 4(2at + b) = 0$$

$$2a - 8at - 4b = 0$$

$$a(2 - 8t) -$$

$c = -at^2 - bt + y_p$

$y_p' = y_p'' t + b$

$$2ae^t + (at+b)e^t - 2$$

$$c_1(\cos t + i\,\mathrm{sen}\,t)\begin{pmatrix}1-i\\1\end{pmatrix} + c_2(\cos t - i\,\mathrm{sen}\,t)\begin{pmatrix}1+i\\1\end{pmatrix}$$

$$\left(c_1\cos t + c_1 i\,\mathrm{sen}\,t - c_1 i\cos t - c_1 i^2\,\mathrm{sen}\,t + c_2\cos t - c_2 i\,\mathrm{sen}\,t + c_2 i\cos t - c_2 i^2\,\mathrm{sen}\,t\right.$$

$$\left(c_1\cos t + c_1 i\,\mathrm{sen}\,t + c_2\cos t - c_2 i\,\mathrm{sen}\,t\right.$$

$$\cos t\,(c_1 + c_2) + \mathrm{sen}\,t\,(c_1 + c_2) - i\,\mathrm{sen}\,t\,(-c_1 + c_2) + i\cos t\,(-c_1 + c_2)$$

$$\cos t\,(c_1 + c_2) - i\,\mathrm{sen}\,t\,(-c_1 + c_2)$$

$$(c_1 + c_2)\begin{pmatrix}\cos t + \mathrm{sen}\,t\\ \cos t\end{pmatrix} + i\,(c_2 - c_1)\begin{pmatrix}\cos t - \mathrm{sen}\,t\\ -\mathrm{sen}\,t\end{pmatrix}$$